Pearson Edexcel

Mark Scheme (Results)

January 2022

Pearson Edexcel International A Level
In Pure Mathematics P3 (WMA13) Paper 01

Question Number	Scheme	Marks
1	$y=(2x+5)e^{3x}$	
	$\frac{dy}{dx}=2e^{3x}+(2x+5)3e^{3x}$	M1 A1
	$\frac{dy}{dx}=0\Rightarrow 2+3(2x+5)=0\Rightarrow x=-\frac{17}{6}$	dM1A1
		(4)
		(4 marks)

M1: Attempts the product rule and achieves a form $\left(\frac{dy}{dx}\right)=Ae^{3x}+B(2x+5)e^{3x}$ where A and B are positive constants. Condone missing/poor bracketing

Note that this could be attempted from $y=2xe^{3x}+5e^{3x}\Rightarrow\frac{dy}{dx}=Ae^{3x}+Bxe^{3x}+Ce^{3x}$

A1: Correct $\frac{dy}{dx}=2e^{3x}+(2x+5)3e^{3x}$ o.e. There is no requirement to simplify this

dM1: Dependent upon the previous M, it is for using a correct strategy to find a value for x.

E.g. Sets or implies $\frac{dy}{dx}=0$, cancels or factorises out the e^{3x} term and solves a linear equation in x

In most cases where lns are used it will be M0

Example of M0

$e^{3x}(6x+17)=0\Rightarrow\ln\left(e^{3x}(6x+17)\right)=\ln(0)\Rightarrow 3x(6x+17)=...$

Example of M1

$e^{3x}(6x+17)=0\Rightarrow(6x+17)=0,\qquad\left(e^{3x}=0\right)$

$x=...$

A1: $x=-\frac{17}{6}$ o.e. only. If an extra solution is given, e.g. from $e^{3x}=0$, it is A0

Condone awrt -2.83 following a correct equation. Ignore any attempt to find y

Question Number	Scheme	Marks
2 (a)	$8\cos\theta = 3\operatorname{cosec}\theta$	
	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1
	Attempts to use $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	$\sin 2\theta = \dfrac{3}{4}$ cso	A1
		(3)
(b)	Correct order of operations to find θ, Look for $\dfrac{\arcsin\left(\frac{3}{4}\right)}{2}$	M1
	$(\theta =)$ awrt $24.3°$	A1
		(2)
		(5 marks)

(a)

B1: States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$. May be implied by the line $8\sin\theta\cos\theta = 3$.

Also allow with a consistent use of a different variable E.g. $\operatorname{cosec} x = \dfrac{1}{\sin x}$ but not $\operatorname{cosec} = \dfrac{1}{\sin}$ unless it has been recovered. Note that $3\operatorname{cosec}\theta = \dfrac{1}{3\sin\theta}$ is B0 unless there is an aside that does state $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

M1: Attempts to use the identity $\sin 2\theta = 2\sin\theta\cos\theta$ and proceeds to $\sin 2\theta = k$ where $|k| \leqslant 1$

Note that an attempt such as $8\sin\theta\cos\theta = 3 \Rightarrow 8\sin 2\theta = 3$ would be M0

A1: Achieves $\sin 2\theta = \dfrac{3}{4}$ o.e. with no errors. Condone the odd notational slip, e.g $\sin\theta \leftrightarrow \sin$

(b)

M1: Uses the correct order of operations to find any value of θ, in degrees or radians that works for their

$\sin 2\theta = k$ follow through on their k with $|k| \leqslant 1$.

In general look for a value of θ found from evaluating $\theta = \dfrac{\arcsin\left(\frac{3}{4}\right)}{2}$.

From a correct equation it is implied by $\theta =$ awrt 0.42 (radians)

A1: $\theta =$ awrt $24.3°$ ONLY

Question Number	Scheme	Marks
3 (i)	$\int (2x-5)^7 \, dx = \frac{(2x-5)^8}{16} + c$	M1 A1
		(2)
(ii)	$\int \frac{4\sin x}{1+2\cos x} \, dx = -2\ln(1+2\cos x) \quad (+c)$	M1 A1
	$\int_0^{\frac{\pi}{3}} \frac{4\sin x}{1+2\cos x} \, dx = \left[-2\ln(1+2\cos x)\right]_0^{\frac{\pi}{3}} = -2\ln 2 + 2\ln 3 = \ln\frac{9}{4}$	dM1 A1
		(4)
		(6 marks)

(i)

M1: Achieves $a(2x-5)^8$ or equivalent where a is a constant. Alternatively achieves au^8 with $u=(2x-5)$

Allow this mark from a miscopy such as $\int (2x-3)^7 \, dx = k(2x-3)^8$

A1: Achieves $\frac{(2x-5)^8}{16} + c$ or exact simplified equivalent such as $\frac{1}{16}(2x-5)^8 + c$. **The +*c* (o.e) must be present.**

Any attempts that start by multiplying out $(2x-5)^7$ are likely to end in failure. They are unlikely to get an expression of the form $a(2x-5)^8$

FYI

$(2x-5)^7 = 128x^7 - 2240x^6 + 16800x^5 - 70000x^4 + 175000x^3 - 262500x^2 + 218750x - 78125$

Score B1 SC for at least 5 out of 8 **correct** terms of

$\frac{128}{8}x^8 - \frac{2240}{7}x^7 + \frac{16800}{6}x^6 - \frac{70000}{5}x^5 + \frac{175000}{4}x^4 - \frac{262500}{3}x^3 + \frac{218750}{2}x^2 - 78125x$

(ii)

M1: Achieves $b\ln(1+2\cos x)$ or $b\ln|1+2\cos x|$ where b is a constant. Condone a missing bracket

Alternatively achieves $b\ln u$ with $u = 1+2\cos x$ (You may see $b\ln ku$ which is also correct)

A1: Achieves $-2\ln(1+2\cos x)$, $-2\ln|1+2\cos x|$ or $-2\ln u$ o.e .with $u=1+2\cos x$ oe. There is no need for $+c$ This may be left unsimplified. Only condone a missing bracket if subsequent work implies one

dM1: Substitutes **both** 0 and $\frac{\pi}{3}$ into an expression of the form $b\ln(1+2\cos x)$ a or $b\ln|1+2\cos x|$ and subtracts either way around . There must have been some attempt to evaluate the trig terms

Alternatively substitutes both 3 and 2 into an expression of the form $k\ln u$ and subtracts

A1: $\ln\frac{9}{4}$

Note that algebraic integration must be seen here. Candidates using their calculators to just write down $\int_0^{\frac{\pi}{3}} \frac{4\sin x}{1+2\cos x} \, dx = 0.81093... = \ln\frac{9}{4}$ should be awarded 0 marks

Question Number	Scheme	Marks
4	$A=\dfrac{80p\mathrm{e}^{0.15t}}{p\mathrm{e}^{0.15t}+4}$	
(a)	$30=\dfrac{80p}{p+4}$	M1
	$30p+120=80p\Rightarrow p=\dfrac{120}{50}=2.4$ *	A1*
		(2)
(b)	$50=\dfrac{80\times2.4\mathrm{e}^{0.15T}}{2.4\mathrm{e}^{0.15T}+4}\Rightarrow 72\mathrm{e}^{0.15T}=200$	M1 A1
	$\Rightarrow 0.15T=\ln\left(\dfrac{200}{72}\right)\Rightarrow T=6.8$	dM1 A1
		(4)
(c)	80 m^2	B1
		(1)
		(7 marks)

(a)

M1: Sets $A=30$ and $\mathrm{e}^{0.15\times0}=1$ to set up an equation in *p*.

A1*: Achieves $p=2.4$ with no significant (*) errors and with one correct **linear (non fractional)** equation in *p*. * Condone minor slips as long as they are recovered before reaching the given answer.

An example of this would be

$$p+4(30)=80p\Rightarrow 30p+120=80p\Rightarrow 50p=120\Rightarrow p=2.4$$

Alt method

M1: Sets $p=2.4$, $\mathrm{e}^{0.15\times0}=1$ and attempts the value of $(A)=\dfrac{80\times2.4\times1}{2.4\times1+4}$

A1*: Achieves $A=30$ with no errors and concludes that *p* = 2.4. Condone $30=\dfrac{192}{6.4}$ ✓

(b) Allow $t\leftrightarrow T$ here

M1: Sets $A=50$, $p=2.4$ and proceeds to an equation of the form $c\mathrm{e}^{0.15t}=d\quad c\times d>0$

Condone slips, e.g. on the 0.15. You may see $d\mathrm{e}^{-0.15t}=c\quad c\times d>0$

A1: Achieves $72\mathrm{e}^{0.15t}=200$ o.e.

dM1: Correct order of operations to find *T/t*

For example $0.15T=\ln\left(\dfrac{200}{72}\right)\Rightarrow T=...$ or

$\ln72+0.15T=\ln200\Rightarrow 0.15T=....\Rightarrow T=...$

A1: AWRT 6.8

(c)

B1: Requires units as well. 80 m^2

Students lacking work in part (b)

Example 1: $50=\dfrac{80\times2.4\mathrm{e}^{0.15T}}{2.4\mathrm{e}^{0.15T}+4}\Rightarrow T=awrt\ 6.8$ can be awarded SC 1000

Example 2: $50 = \dfrac{80 \times 2.4e^{0.15T}}{2.4e^{0.15T} + 4} \Rightarrow 72e^{0.15T} = 200 \Rightarrow T = awrt\ 6.8$ score M1 A1 via scheme and then

SC 10

Question Number	Scheme	Marks
5 (a)	Attempts to find y at -1.25 and -1.2 with one correct to 1sf	M1
	Achieves $y(-1.25) = -0.9$ and $y(-1.2) = 0.2$	
	With reason (change of sign and continuous) and Conclusion	A1
		(2)
(b)	(i) Attempts $\sqrt{12\ln(15)+8}$	M1
	$x_2 = \text{awrt } 6.3637$	A1
	(ii) $x = 6.4142$	B1
		(3)
(c)	$\dfrac{dy}{dx} = \dfrac{12}{2x+3} - x$	M1 A1
	Stationary point when $\dfrac{12}{2x+3} = x \Rightarrow 2x^2 + 3x - 12 = 0 \Rightarrow x =$	dM1
	$\Rightarrow x = \dfrac{-3+\sqrt{105}}{4}$ or awrt 1.81 ONLY	A1
		(4)
		(9 marks)

(a)

M1: Attempts to find y at -1.25 and -1.2 with one correct to at least 1sf.

FYI $y(-1.25) = -0.9$ and $y(-1.2) = 0.2$

A1: Achieves $y(-1.25) = \text{awrt } -0.9$ and $y(-1.2) = \text{awrt } 0.2$ with a reason and minimal conclusion.

Acceptable reasons are; " sign change and continuous", " $y(-1.25) \times y(-1.2) < 0$ and continuous" Minimal conclusions are; "hence proven", "hence root", "✓", "☐"

Note: A smaller interval could be chosen but it must span -1.20998 and the final conclusion must refer to the given interval to score both marks.

(b)(i)

M1: Attempts to apply the iteration formula once. Accept $\sqrt{12\ln(15)+8} = ...$ or awrt 6.4

A1: $x_2 = \text{awrt } 6.3637$

(b)(ii)

B1: CAO $x = 6.4142$ There must be some evidence of the M or continued iteration for this to be awarded. A minimum would be an attempt at x_2 (the M) or an attempt at any intermediate term.

(c)

M1: Attempts to differentiate with $\ln(2x+3) \to \dfrac{...}{2x+3}$

A1: $\dfrac{dy}{dx} = \dfrac{12}{2x+3} - x$ which may be left unsimplified . No requirement to see lhs

dM1: Sets their $\dfrac{...}{2x+3} \pm ...x = 0$ and proceeds to a value for x via a correct method of solving a 3TQ

There must be some evidence of working but allow candidates to use a calculator to write down the solution of 3TQ. If they do, it must be correct for their 3TQ

A1: cso $x=\dfrac{-3+\sqrt{105}}{4}$ or awrt 1.81 ONLY. If $x=\dfrac{-3-\sqrt{105}}{4}$ or awrt -3.31 is also written down it must be rejected. ISW after a correct answer. There must be evidence of dM1 to award this mark

Question Number	Scheme	Marks
6. (a)	$\mathrm{f}(x)=\dfrac{5x-3}{x-4}\Rightarrow \mathrm{f}'(x)=\dfrac{5(x-4)-(5x-3)}{(x-4)^2}=\dfrac{k}{(x-4)^2}$	M1 dM1
	States that $\mathrm{f}'(x)=\dfrac{-17}{(x-4)^2}\Rightarrow \mathrm{f}'(x)<0$ hence decreasing * cso	A1*
		(3)
(b)	$y=\dfrac{5x-3}{x-4}\Rightarrow xy-4y=5x-3\Rightarrow xy-5x=4y-3$	M1
	$\Rightarrow x=\dfrac{4y-3}{y-5}$ So $\mathrm{f}^{-1}(x)=\dfrac{4x-3}{x-5}$	A1
	Domain $x>5$	B1
		(3)
(c) (i)	$\mathrm{ff}(x)=\dfrac{5\times\dfrac{5x-3}{x-4}-3}{\dfrac{5x-3}{x-4}-4}$	M1
	$\mathrm{ff}(x)=\dfrac{5\times(5x-3)-3(x-4)}{5x-3-4(x-4)}=\dfrac{22x-3}{x+13}$	dM1 A1
(ii)	$5<\mathrm{ff}(x)<22$	B1, B1
		(5) **(11 marks)**

(a)

M1: Attempts to use the quotient rule to achieve a form $\dfrac{p(x-4)-q(5x-3)}{(x-4)^2}$ with $p,q>0$ OR attempts the product rule to achieve a form $p(x-4)^{-1}\pm(5x-3)(x-4)^{-2}$

Condone attempts in which terms such as $(x-4)^2$ may have been multiplied out incorrectly

dM1: Proceeds to $\mathrm{f}'(x)=\dfrac{k}{(x-4)^2}$ Allow the lhs to be $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

A1*: Requires a correct $\mathrm{f}'(x)$, a correct statement such as "$\mathrm{f}'(x)<0$" o.e (such as $\mathrm{f}'(x)$ is negative) and a minimal conclusion which could be ✓ or QED or ☐

It cannot be awarded from substituting in single values of *x*. It cannot be based on incorrect mathematics.

"There is a minus sign" or "$-17<0$" is insufficient without the further statement $\mathrm{f}'(x)<0$ (o.e) followed by a minimal conclusion.

(b)

M1 Attempts to change the subject. Look for cross multiplication with an attempt to collect terms with *x*'s (or replaced *y*'s) on one side of the equation and non *x* terms on the other side

A1 $\mathrm{f}^{-1}(x)=\dfrac{4x-3}{x-5}$ o.e Condone the lhs as f^{-1} and even y. The notation $\mathrm{f}^{-1}:x\mapsto\dfrac{4x-3}{x-5}$ is fine

B1 Correct domain $x > 5$

(c) (i)

M1 Attempts to substitute $\frac{5x-3}{x-4}$ into f. Condone minor slips but the form of the expression must be correct

dM1 Multiplies all terms on numerator and denominator by $x-4$. Condone missing brackets

Alternatively writes both the numerator and denominator as single fractions over $(x-4)$

A1 $(\text{ff}(x)) = \frac{22x-3}{x+13}$ Condone a missing left hand side or being set as $y =$

(c)(ii)

B1: Achieves the value of one "end". It is not just for the number so f f < 5 is B0

Condone non strict inequalities here so $5 \leqslant \text{ff} < 21$ would be fine for B1 B0

B1: Fully correct. Accept $5 < \text{ff} < 22$, $(5, 22)$. Allow with y or range but $5 < x < 22$ would be B1 B0

Methods where candidates "split up" fraction $\frac{5x-3}{x-4} = 5 + \frac{17}{x-4}$

(a)

M1:Attempts to split into form $5 + \frac{17}{x-4}$ AND attempts to use the chain rule.

Look for $\frac{5x-3}{x-4} \to A + \frac{B}{x-4}$ with a differential of $\frac{k}{(x-4)^2}$

dM1: Proceeds to $\text{f}'(x) = \frac{k}{(x-4)^2}$ Allow the lhs to be $\frac{\text{d}y}{\text{d}x}$

A1*: $\text{f}'(x) = \frac{-17}{(x-4)^2}$ and states $\text{f}'(x) < 0$ (for all x) so f is decreasing function

(b)

M1: Attempts to split $y = \frac{5x}{x+3}$ into form $y = 5 + \frac{17}{x-4}$ and then attempts to make x the subject.

Look for $y = \frac{5x-3}{x-4} = A + \frac{B}{x-4}$ with progress to a form $x - 4 = \frac{B}{\pm y \pm A}$

A1: $\text{f}^{-1}(x) = \frac{17}{x-5} + 4$ or equivalent

B1: $x > 5$

(c)

M1: Attempts to split $y = \frac{5x}{x+3}$ into form $y = 5 + \frac{17}{x-4}$ and then attempts $\text{ff}(x) = 5 + \frac{17}{5 + \frac{17}{x-4} - 4}$

dM1: Uses a correct method to combine into a single fraction of the required form

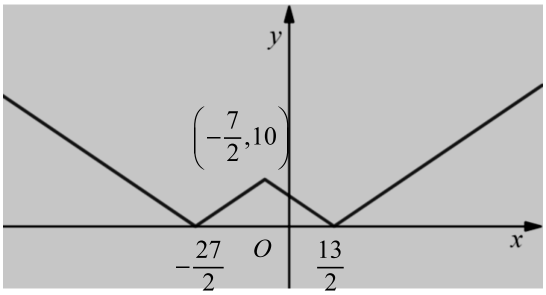
A1: $\text{ff}(x) = \frac{22x-3}{x+13}$

(c)(ii)

B1: Achieves the value of one "end". It is not just for the number so ff < 5 is B0

Condone non strict inequalities here so $5 \leqslant \text{ff} < 21$ would be fine for B1 B0

B1: Fully correct. Accept $5 < \text{ff} < 22$, $(5,22)$. Allow with y or range but $5 < x < 22$ would be B1 B0

Question Number	Scheme	Marks
7 (a)	$\left(-\frac{7}{2}, -10\right)$	B1 B1
		(2)
(b)	Attempts to solve either $\frac{1}{2}(2x+7)-10...\frac{1}{3}x+1$	
	Or $-\frac{1}{2}(2x+7)-10...\frac{1}{3}x+1$	M1
	Both correct critical values $x...-\frac{87}{8}, \frac{45}{4}$	A1
	Selects outside region for their critical values	dM1
	Correct solution $x \leqslant -\frac{87}{8}, x \geqslant \frac{45}{4}$	A1
		(4)
(c)	 Shape	B1
	Maximum	B1 ft
	One correct minimum	B1
	Fully correct	B1
		(4)
		(10 marks)

(a) Must be answered in (a). We cannot imply from work in (c)

B1: One correct coordinate. Allow as $x = -\frac{7}{2}, y = -10$. Allow exact equivalents , e.g. $x = -\frac{14}{4}$

B1: Both coordinates correct. Allow as $x = -\frac{7}{2}, y = -10$. Allow exact equivalents , e.g. $x = -\frac{14}{4}$

(b)

M1: Attempts to solve a correct equation or inequality. They must proceed as far as x ...

Condone slips where they attempt to change the subject in an attempt to make $\left|2x+7\right|$ the subject.

So either of $\frac{1}{2}\left|2x+7\right|-10...\frac{1}{3}x+1 \Rightarrow \left|2x+7\right| = ax+b \Rightarrow 2x+7 = ax+b$

or $\frac{1}{2}\left|2x+7\right|-10...\frac{1}{3}x+1 \Rightarrow \left|2x+7\right| = ax+b \Rightarrow -2x-7 = ax+b$ are condoned for the M1

A1: Correct critical values $x...-\frac{87}{8}, \frac{45}{4}$ which may be part of an incorrect inequality

dM1: Selects outside region for their critical values. Allow these to be written as one inequality. Allow the strict inequalities here. It is dependent upon having attempted to solve **one** correct equation

A1: $x \leqslant -\frac{87}{8}, x \geqslant \frac{45}{4}$ These can be given separately but do not isw here. Condone words like "and" between. Allow other correct forms such as $x \in \left(-\infty, -\frac{87}{8}\right] \cup \left[\frac{45}{4}, \infty\right)$ **Mark their final answer**

Do not allow incorrect answers such as $\frac{45}{4} \leqslant x \leqslant -\frac{87}{8}$ or $x \in \left(-\infty, -\frac{87}{8}\right] \cap \left[\frac{45}{4}, \infty\right)$

(c) There must be a sketch for this. If the text and the graph contradict each other the graph takes precedence

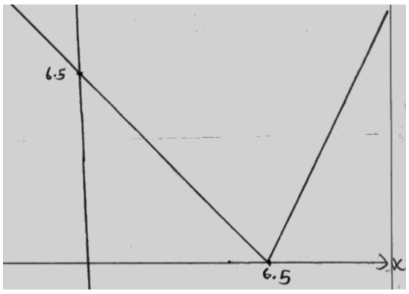
B1: W shape any position. Condone asymmetric arms.

B1ft: Maximum point but follow through on their coordinates from part (a)

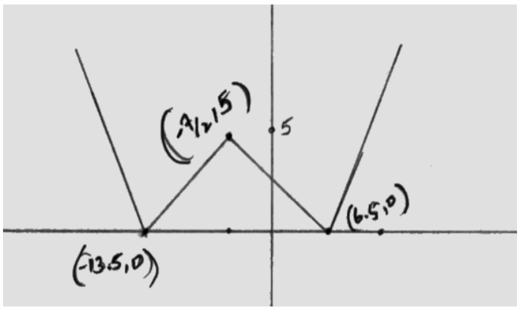
B1: Either minimum value. It must be minimum point and not just the point at which the graph crosses the x-axis. Condone $(0, 6.5)$ for $(6.5, 0)$ as long as it is on the correct axis

B1: Fully correct diagram with **ALL** points correct (allow exact equivalents) Condone asymmetric arms.

Accept a W on the x axis with both $x = \frac{13}{2}$ and $x = -\frac{27}{2}$ marked. Coordinates must be correct



B0 B0 B1 (one local minimum correct) B0

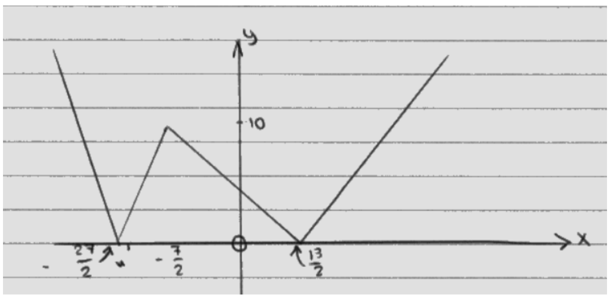


B1: W shape

B1ft: Their part (a) was $\left(-\frac{7}{2}, -5\right)$

B1: One correct local minimum. In fact both are correct

B0: Must be completely correct. Incorrect maximum point



Acceptable for all marks. It is a little asymmetric and the local maximum point is implied

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You may see attempts at part (b) via squaring. It is scored in a similar way.

Any attempt must attempt to isolate the modulus term before squaring

$$\frac{1}{2}|2x+7| - 10 \ldots \frac{1}{3}x + 1 \Rightarrow \frac{1}{2}|2x+7| \ldots \frac{1}{3}x + 11$$

M1: Squares and attempts to solve $\frac{1}{4}(2x+7)^2 \ldots \left(\frac{1}{3}x+11\right)^2$ via a quadratic

A1: FYI Quadratic is $32x^2 - 12x - 3915$ which has critical values of $-\frac{87}{8}$ and $\frac{45}{4}$ which may be found via a calculator

dM1: Selects outside region. It is dependent upon a correct $\frac{1}{4}(2x+7)^2 ... \left(\frac{1}{3}x+11\right)^2$ o.e.

A1: $x \leqslant -\frac{87}{8}, x \geqslant \frac{45}{4}$ Allow other forms such as $x \in \left(-\infty, -\frac{87}{8}\right] \cup \left[\frac{45}{4}, \infty\right)$

Question Number	Scheme	Marks
8 (a)	$\log_{10} x = 2.74 - 0.079t$	
	$\log_{10} x = 2.74 - 0.079t$; $x = 10^{2.74} \times 10^{-0.079t}$; $x = 10^{2.74} \times \left(10^{0.079}\right)^{-t}$ | $x = p\,q^{-t}$; $\log_{10} x = \log_{10} pq^{-t}$; $\log_{10} x = \log_{10} p - t\log_{10} q$	
	A correct equation for p or q. E.g. either $\log_{10} p = 2.74$ or $\log_{10} q = 0.079$	M1
	A correct value for either p or q. Either $p = \text{awrt } 550$ or $q = \text{awrt } 1.2$	A1
	A correct equation for both p and q. E.g both $p = 10^{2.74}$ and $q = 10^{0.079}$	dM1
	Both $p = \text{awrt } 550$ and $q = \text{awrt } 1.2$ **with** proof*	A1*
		(4)
(b)	"p" is the amount of antibiotic (in mg) in the patient's bloodstream at the start	B1
		(1)
(c)	$x = 400 \times 1.4^{-t} \Rightarrow \frac{dx}{dt} = -400\ln 1.4 \times 1.4^{-t}$	B1
	Substitutes $t = 5$ into $\frac{dx}{dt} = \text{awrt } -25$	M1 A1
		(3)
		(8 marks)

(a)
M1:A correct equation in either p or q

For p look for $\log_{10} p = 2.74$ or $p = 10^{2.74}$ For q look for $\log_{10} q = 0.079$ or $q = 10^{0.079}$

This is implied by a correct value for either p or q (See below), $x = 550 \times q^{-t}$, or $x = p \times 1.2^{-t}$

A1:Either $p = \text{awrt } 550$, $q = \text{awrt } 1.2$, $x = 550 \times q^{\pm t}$, or $x = p \times 1.2^{-t}$

dM1:A correct equation for both p and q. E.g. sight of both $p = 10^{2.74}$ and $q = 10^{0.079}$

This may be implied by $x = 550 \times 1.2^{-t}$

A1*: Both $p = \text{awrt } 550$ and $q = \text{awrt } 1.2$ **with** proof* as it is a "show that" question.

There must be no incorrect working. Correct values of p and q are implied by $x = 550 \times 1.2^{-t}$
This "proof" part can be shown with a minimum of working

Starting with $x = p\,q^{-t}$ a minimum of $\log_{10} x = \log_{10} p - t\log_{10} q$ followed by correct equations and values is sufficient

Starting with $\log_{10} x = 2.74 - 0.079t$ a minimum of $x = 10^{2.74} \times 10^{-0.079t}$ followed by correct equations and/or values is sufficient

(b)

B1 Award for a statement that refers or alludes to the amount of antibiotic in the bloodstream when t = 0

E.g. "p" is the amount of antibiotic (in mg) (in the patient's bloodstream) at the start

Condone that "p" is the dose of antibiotic (in mg) given to the patient

Condone " just / immediately after" but not "before" the dose is given

Don't allow students to give a correct and an incorrect answer. This would be B0

(c)

B1: $x = 400 \times 1.4^{-t} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = \pm 400 \ln 1.4 \times 1.4^{-t}$

M1: Substitutes $t = 5$ into a changed function of the form $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) = k \times 1.4^{-t}$ where $k \neq 400$

The left hand side may be incorrect. E.g. $\frac{\mathrm{d}y}{\mathrm{d}x}$ or similar

A1: $\frac{\mathrm{d}x}{\mathrm{d}t} = \text{awrt} -25$

Note that candidates who lose the negative sign when differentiating to get $\frac{\mathrm{d}x}{\mathrm{d}t} = \text{awrt } 25$ score B1 M1 A0

Alt seen;

B1: $x = 400 \times 1.4^{-t} \Rightarrow \ln x = \ln 400 - t \ln 1.4 \Rightarrow \frac{1}{x}\frac{\mathrm{d}x}{\mathrm{d}t} = \pm \ln 1.4$

M1 A1: As before

Question Number	Scheme	Marks
9 (i)	$2\sec^2 x - 3\tan x = 2$	
	$2\left(1+\tan^2 x\right) - 3\tan x = 2$	M1
	$2\tan^2 x = 3\tan x \Rightarrow \tan x = \frac{3}{2}, (\tan x = 0)$	dM1
	$x = \text{awrt } 0.983, \pi$	A1, B1
		(4)
(ii)	$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} \equiv \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta} \equiv \frac{\sin(3\theta-\theta)}{\sin\theta\cos\theta}$	M1
	$\equiv \frac{\sin(3\theta-\theta)}{\frac{1}{2}\sin 2\theta}$	dM1, A1
	$\equiv \frac{\cancel{\sin 2\theta}}{\frac{1}{2}\cancel{\sin 2\theta}} \equiv 2$ *	A1*
		(4)
		(8 marks)

(i)

M1: Attempts to use $\sec^2 x = \pm 1 \pm \tan^2 x$ within the given equation, condoning slips

There are alternatives using for example $\sec^2 x = \frac{1}{\cos^2 x}$, $\tan x = \frac{\sin x}{\cos x}$ and $\cos^2 x = 1 - \sin^2 x$

FYI the correct equation using this method would be $2\sin^2 x - 3\sin x\cos x = 0$

dM1: Valid method of solving a quadratic equation in $\tan x$. Condone division by $\tan x$

In the alternative the method of solving is similar $\sin x(2\sin x - 3\cos x) = 0 \Rightarrow \tan x = \frac{3}{2}$

A1: $x = \text{awrt } 0.983$ as the only solution to $\tan x = \frac{3}{2}$ in the region $(0, \pi]$

B1: $x = \pi$ or awrt 3.14 (as the only solution in the range for $\tan x = 0$ or $\sin x = 0$ in the region $(0, \pi]$).

Condone the inclusion of 0 as that is outside the range

Condone 180° if both angles have been given in degrees. Ignore any solution outside the range

(ii)

M1: Attempts to form a single fraction and uses

- either the compound angle formula on the numerator. Allow $\sin(3\theta \pm \theta)$
- or the double angle on the denominator. Allow $\sin\theta\cos\theta = k\sin 2\theta$

dM1: Attempts to form a single fraction and uses both

- the compound angle formula on the numerator. Must be $\sin(3\theta - \theta)$
- and the double angle on the numerator or denominator. Allow $\sin\theta\cos\theta = k\sin 2\theta$

A1: For reaching the form $\frac{\sin(3\theta-\theta)}{\frac{1}{2}\sin 2\theta}$ **or** $\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$

A1*: Fully complete proof.

Alt soln

M1: Attempts compound angle formulae for both $\sin 3\theta$ and $\cos 3\theta$. Condone **only** sign slips

$$\frac{\sin 3\theta}{\sin\theta}-\frac{\cos 3\theta}{\cos\theta}=\frac{\sin(2\theta+\theta)}{\sin\theta}-\frac{\cos(2\theta+\theta)}{\cos\theta}=\frac{\sin 2\theta\cos\theta+\cos 2\theta\sin\theta}{\sin\theta}-\frac{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}{\cos\theta}$$

dM1: This mark is dependent upon the previous one having been awarded. It can be awarded for one of

- attempting to use the double angle formula for $\sin 2\theta$ (condone $\sin 2\theta = k\sin\theta\cos\theta$ if used consistently), then dividing and cancelling out the terms in $\cos 2\theta$ to produce an expression in just $a\cos^2\theta \pm b\sin^2\theta$

  E.g. $\frac{2\cancel{\sin\theta}\cos\theta\cos\theta}{\cancel{\sin\theta}}+\frac{\cos 2\theta\cancel{\sin\theta}}{\cancel{\sin\theta}}-\frac{\cos 2\theta\cancel{\cos\theta}}{\cancel{\cos\theta}}\pm\frac{2\sin\theta\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}}=....$

- as above but may use $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ o.e. to produce an expression in just $a\cos^2\theta + b\sin^2\theta + c$

  E.g.

$$\frac{2\cancel{\sin\theta}\cos\theta\cos\theta}{\cancel{\sin\theta}}+\frac{(\cos^2\theta-\sin^2\theta)\cancel{\sin\theta}}{\cancel{\sin\theta}}-\frac{(\cos^2\theta-\sin^2\theta)\cancel{\cos\theta}}{\cancel{\cos\theta}}\pm\frac{2\sin\theta\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}}=...$$

- Uses appropriate expressions for $\sin 2\theta$ and $\cos 2\theta$ then uses a correct common factor to produce a single fraction in just $\sin\theta$ and $\cos\theta$

E.g.

$$\frac{2\sin\theta\cos\theta\cos^2\theta+(\cos^2\theta-\sin^2\theta)\sin\theta\cos\theta}{\sin\theta\cos\theta}-\frac{(\cos^2\theta-\sin^2\theta)\sin\theta\cos\theta\pm 2\sin\theta\cos\theta\sin^2\theta}{\sin\theta\cos\theta}=...$$

A1: For a correct intermediate line in single angles that leads to the given answer
In most methods it is for one of

- $\frac{2\cancel{\sin\theta}\cos\theta\cos\theta}{\cancel{\sin\theta}}+\frac{2\sin\theta\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}}$ or equivalent
- $2\cos^2\theta+1-2\sin^2\theta+1-2\cos^2\theta+2\sin^2\theta$
- $\frac{2\sin\theta\cos^3\theta+2\sin^3\theta\cos\theta}{\sin\theta\cos\theta}$
- $\frac{2\cancel{\sin\theta}\cos\theta\cos\theta+\cos 2\theta\cancel{\sin\theta}}{\cancel{\sin\theta}}-\frac{\cos 2\theta\cancel{\cos\theta}-2\sin\theta\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}}$ or equivalent.

We can see clearly that the $\cos 2\theta$ terms cancel so that it is effectively an expression in single angles

A1*: Fully complete proof with no errors. You may tolerate the odd notational slip.
Withhold this mark if there are obvious and repeated notational errors

....

Most answers seen will be a combination of these.
Generally the marks are awarded for
M1: Makes a positive step towards achieving the given answer
dM1: Makes all the correct steps towards the proof but allow slips
A1: A correct line that usually only requires the use of the pythagorean identity to reach the given answer
If you see the triple angle identities used (and it is incorrect) please send to review

$$\frac{\sin 3\theta}{\sin\theta}-\frac{\cos 3\theta}{\cos\theta}\equiv\frac{3\sin\theta-4\sin^3\theta}{\sin\theta}-\frac{4\cos^3\theta-3\cos\theta}{\cos\theta}\equiv(3-4\sin^2\theta)-(4\cos^2\theta-3)\equiv 6-4(\sin^2\theta+\cos^2\theta)\equiv 2$$

Question Number	Scheme	Marks
10 (a)	$x = y\mathrm{e}^{2y} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2y} + 2y\mathrm{e}^{2y}$	M1, A1
	$\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{x}{y} + 2x$	
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\dfrac{x}{y} + 2x}$	dM1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{x + 2xy} = \dfrac{y}{x(1+2y)}$ *	A1*
		(4)
(b)	Deduces $y = -\dfrac{1}{2}$	B1
	Substitutes $y = -\dfrac{1}{2} \Rightarrow x = -\dfrac{1}{2\mathrm{e}}$ Range for k $-\dfrac{1}{2\mathrm{e}} < k < 0$	M1, A1
		(3)
		(7 marks)

(a)

M1: For attempting to differentiate with respect to y.

Uses the product rule on $y\mathrm{e}^{2y} \Rightarrow \mathrm{e}^{2y} + ...y\mathrm{e}^{2y}$. The left hand side may be missing/incorrect

A1: Correct differentiation E.g. $\dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2y} + 2y\mathrm{e}^{2y}$

dM1: Full method to get $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of just x and y.

This requires, in any order

- a correct attempt to invert, e.g. not inverting each term in a sum of terms
- e^{2y} being fully replaced. You should see e^{2y} being replaced by $\dfrac{x}{y}$ or equivalent and $y\mathrm{e}^{2y}$ being replaced by x

A1*: Correct proof. All relevant steps should be shown and there should be no errors.

If you feel that it hasn't been fully shown then please award M1 A1 dM1 A0

(b) Now being marked B1 M1 A1. On epen it is set up M1 A1 A1

B1: For deducing left hand end occurs when $y = -\dfrac{1}{2}$.

M1: For attempting to find x when $y = -\dfrac{1}{2} \Rightarrow x = ...$ This may be implied by k = awrt -0.183 or -0.184

A1: $-\dfrac{1}{2\mathrm{e}} < k < 0$ or **exact** equivalent such as e.g. $-\dfrac{1}{2}\mathrm{e}^{-1} < k < 0$, $k > -\dfrac{1}{2}\mathrm{e}^{-1}$ **and** $k < 0$, $\left\{k : k > -\dfrac{1}{2\mathrm{e}}\right\} \cap \{k : k < 0\}$

This must be correct so the candidate cannot have two separate inequalities with an "**or**" between

This must be the range for k not x

....
......

Alt via ln's

$$x = y\mathrm{e}^{2y} \Rightarrow \ln x = \ln y + 2y \quad \text{o.e.}$$

M1: Attempts to differentiate wrt x . Consider just rhs $\ln y + 2y \rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} + ...\frac{\mathrm{d}y}{\mathrm{d}x}$

Alternatively attempts to differentiate wrt y . Consider both sides $...\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{y} + 2$

A1: Correct differentiation $\frac{1}{x} = \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} + 2\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{1}{x}\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{y} + 2$

....
....

Alt via differentiating wrt x

$$x = y\mathrm{e}^{2y} \Rightarrow \quad 1 = \mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} + y \times 2\mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$1 = \frac{x}{y}\frac{\mathrm{d}y}{\mathrm{d}x} + 2x\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$1 = \left(\frac{x + 2xy}{y}\right)\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x(1+2y)} \quad *$$

M1: For attempting to differentiate wrt x.

Uses the product rule on $y\mathrm{e}^{2y} \Rightarrow \quad \mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} + ...y\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{e}^{2y}$. The left hand side may be missing/incorrect

Uses the quotient rule on $y = \frac{x}{\mathrm{e}^{2y}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^{2y} - 2x\mathrm{e}^{2y} \times \frac{\mathrm{d}y}{\mathrm{d}x}}{\mathrm{e}^{4y}}$ condoning slips on the coefficient

A1: Correct differentiation including the lhs

dM1: Full method to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.